Essential statistics for quality assurance (I)

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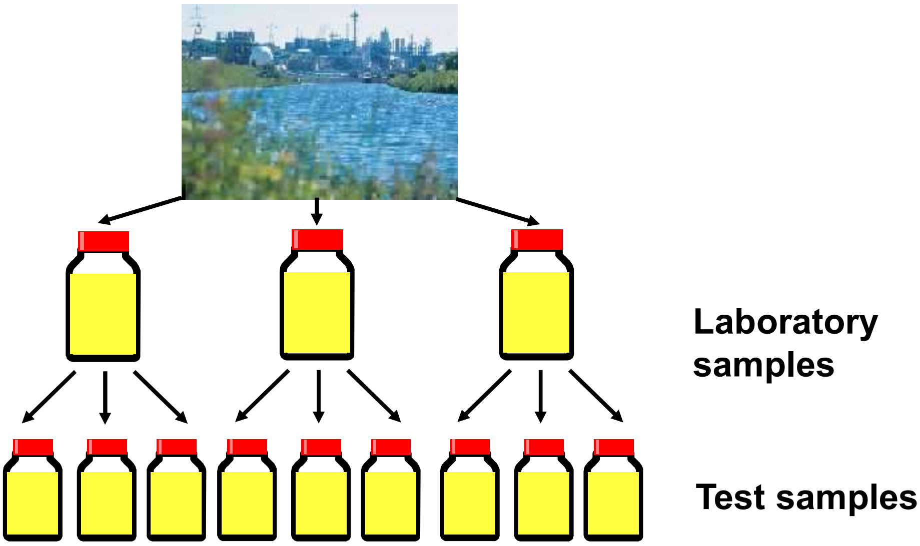
Overview

- Sample vs population statistics
- Properties of the normal distribution
- Summary statistics
  - mean, standard deviation, relative standard deviation, standard deviation of the mean
- Applications of statistics
  - setting limits on control charts
  - interpreting PT scores (*z*-scores)
- Significance testing
  - procedure

Summary statistics

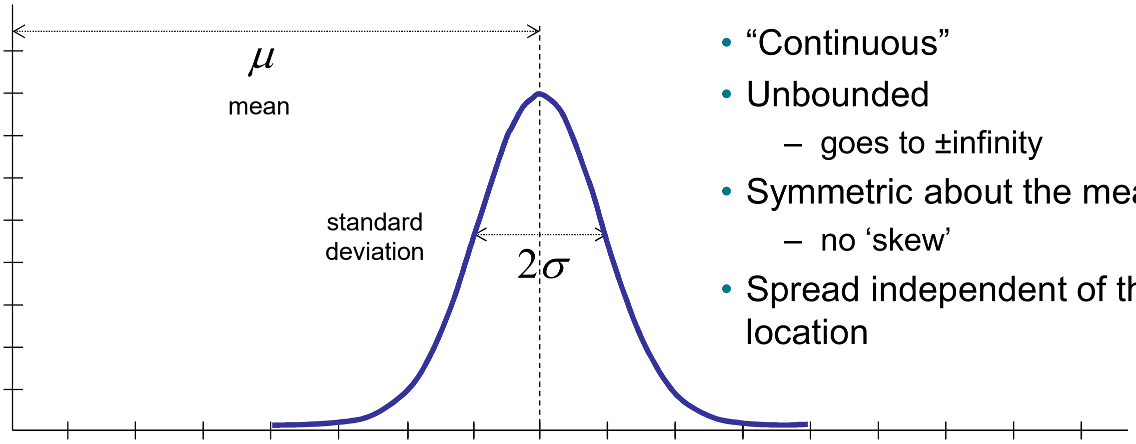
Sample vs population (1)



Sample vs population (2)

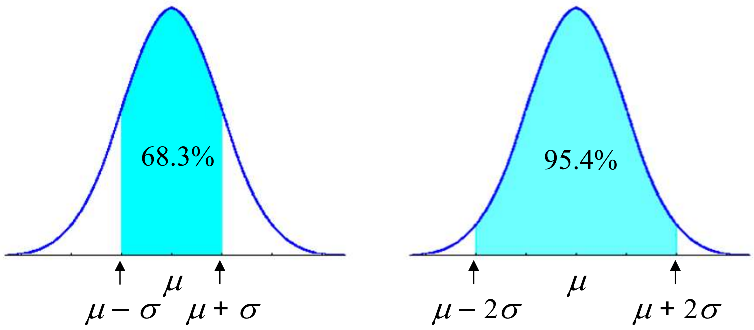
- Laboratories are limited in the number of measurements they can make
- Assume that observations obtained in the laboratory are a random sample from a potentially infinite population
- Population parameters (population mean, population standard deviation)
  - unknown true values of interest
  - represented by Greek alphabet (μ, σ)
- Laboratories use and report 'sample statistics'
  - provide an estimate of the population parameters
  - represented by Latin alphabet ($\bar{x}$, s)

The normal distribution



- "Continuous"
- Unbounded
  - goes to ±infinity
- Symmetric about the mean
  - no 'skew'
- Spread independent of the location

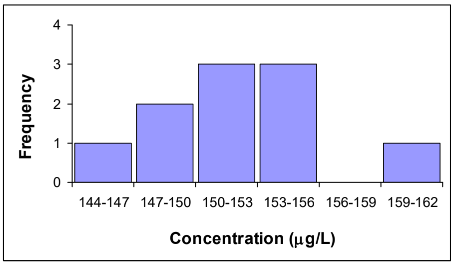
Areas under the normal curve



$\pm\sigma$	% population
1.00	68.3
1.64	90.0
1.96	95.0
2.00	95.4
2.57	99.0
3.00	99.7

Summary statistics

Lead (µg/L)	
152	151
155	145
161	155
151	149
156	150



Sample mean

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = 152.5\ \mu\text{g/L}$$

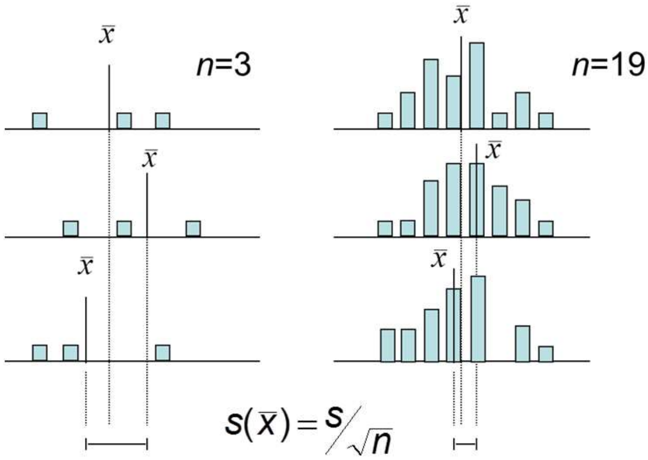
Sample standard deviation

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}} = 4.4\ \mu\text{g/L}$$

%relative standard deviation (coefficient of variation)

$$\%\text{rsd} = \%\text{CV} = \frac{s}{\bar{x}} \times 100 = 2.9\%$$

Standard deviation of the mean



$$s(\bar{x}) = s/\sqrt{n}$$

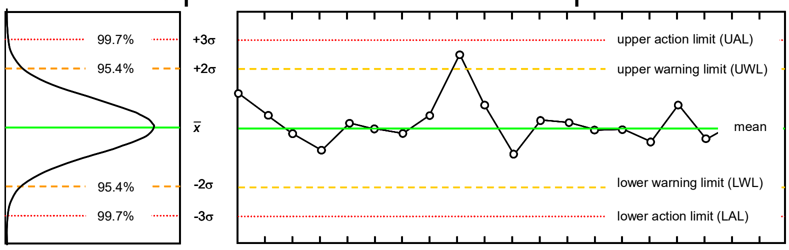
where *s* is the sample standard deviation

Applications of statistics in QC & QA

- Interpretation of quality control results
  - control charts
- Proficiency testing scores

x-chart (Shewhart chart)

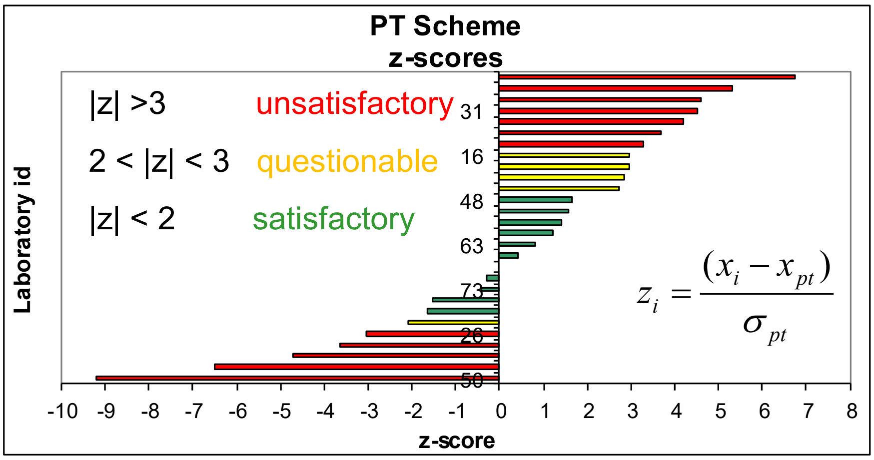
- Used to monitor bias and precision
- Individual control values plotted in time ordered sequence



- Key features
  - central line
  - upper and lower warning limits
  - upper and lower action limits

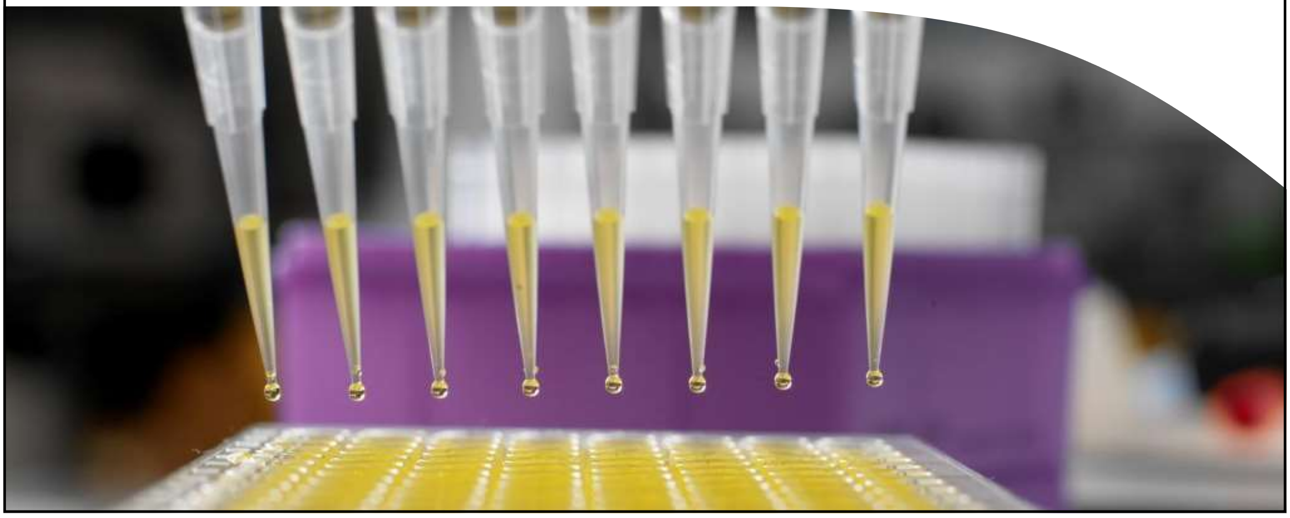
Also known as an 'individuals chart'

Proficiency testing/External quality assessment



x_i laboratory's result
x_{pt} assigned value
σ_{pt} target standard deviation

Principles of significance testing

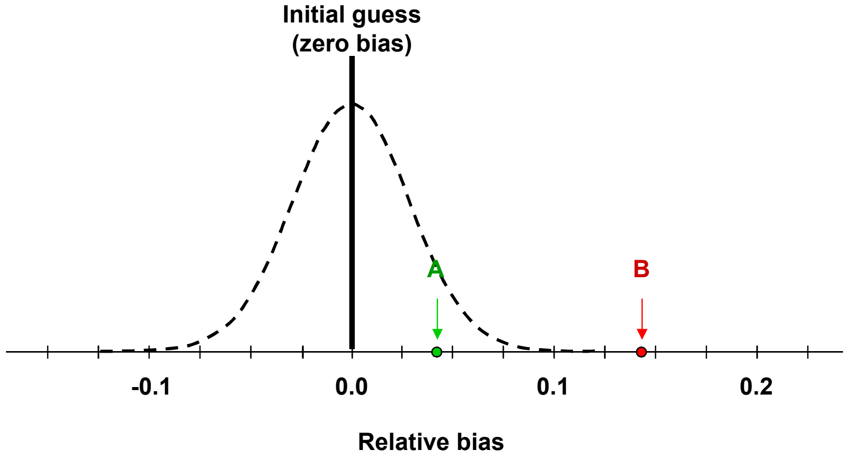


Principles of significance testing

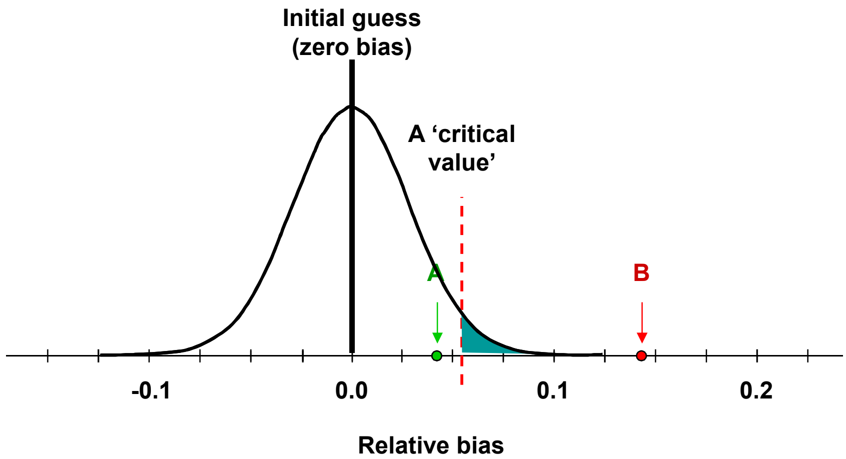
- Make a guess about the true state of affairs (H_0)
  - there is no significant bias/systematic error
  - the precision of two methods is equivalent
  - there are no outliers in a data set
- Ask whether observations are consistent with that guess
  - we calculate the probability that any difference between the observation data and that guess arises solely from random error
- Types of parametric tests
  - *t*-test: Comparing means
  - *F*-test: Comparing variances*
  - analysis of variance (ANOVA): Comparing multiple sets of data

*variance = s^2

Principles of significance testing



Significance and probability

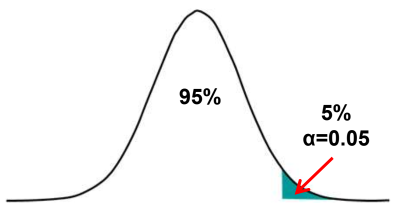


Significance testing procedure

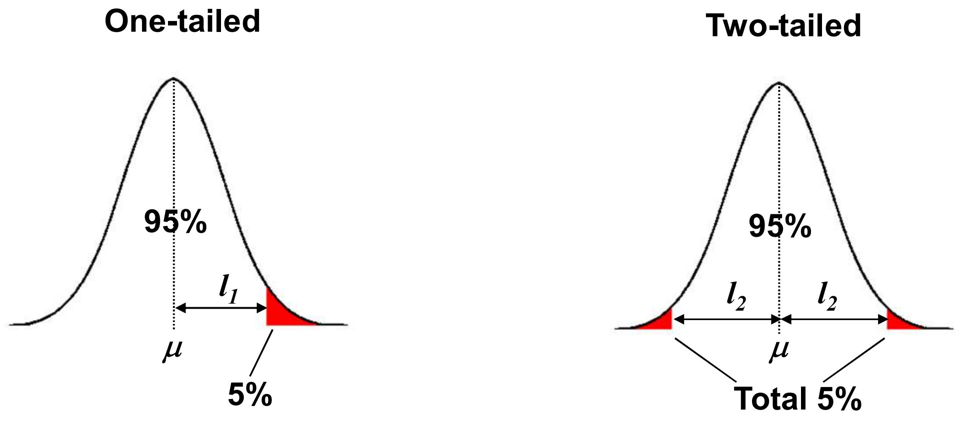
1. State the question/hypothesis
2. Select the appropriate test
3. Choose a level of significance
4. Decide number of tails
5. Calculate degrees of freedom in the data
6. Look up the critical value (tables or software)
7. Calculate the test statistic from the data
8. Compare test statistic with critical value (or use p-values)

Choose the level of significance

- Level of significance is related to probability
- 5% significance level is appropriate for most uses
- Corresponds to 5% (0.05) probability of making the wrong decision
- A 5% significance level corresponds to a confidence level of 95%



Decide the number of tails



Hypotheses and tails

i) "The mean is equal to the given value" ($\mu = x_0$)
vs "The mean is *less than* the given value" ($\mu < x_0$)
one-tailed test

ii) "The mean is equal to the given value" ($\mu = x_0$)
vs "The mean is *greater than* the given value" ($\mu > x_0$)
one-tailed test

iii) "The mean is equal to the given value" ($\mu = x_0$)
vs "The mean is *not equal to* the given value" ($\mu \neq x_0$)
two-tailed test

Finding the critical value

- Critical values for *t*- and F-tests are found from tables or statistical software
- Need to know the
  - significance level
  - degrees of freedom
  - number of tails

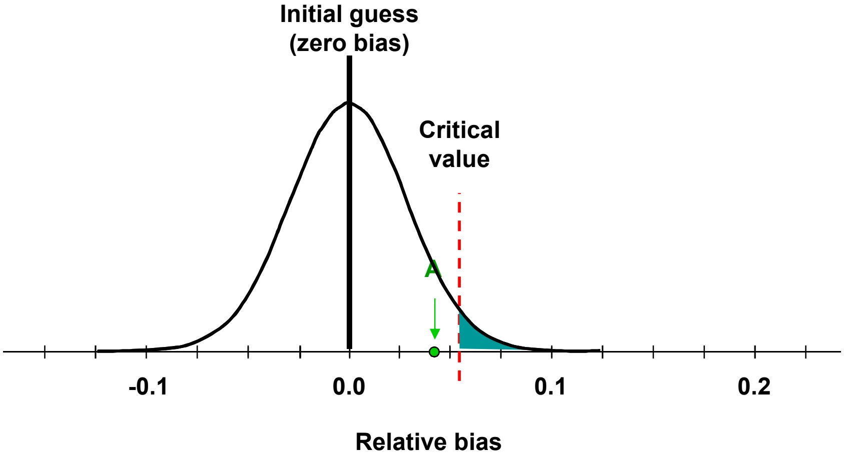
Partial 2-tailed *t*-table

ν	**95%**	**99%**
1	12.7	63.7
2	4.3	9.9
3	3.2	5.8
5	2.6	4.0
9	2.3	3.3
∞	1.96	2.58

Carrying out the test

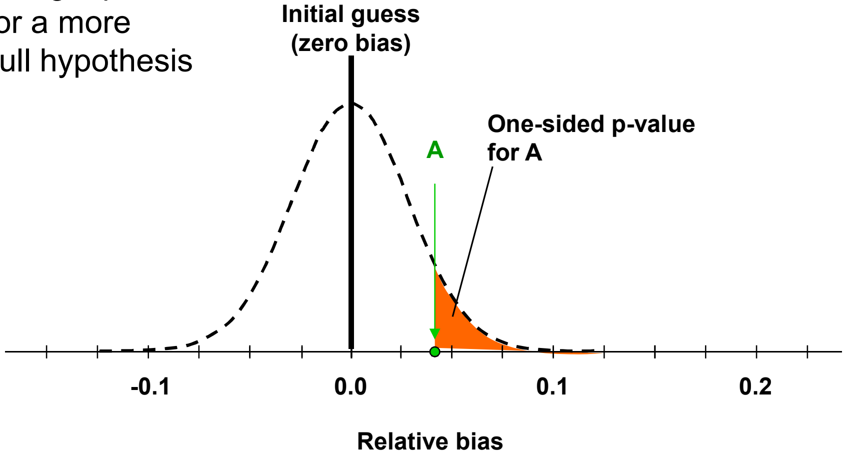
- Calculate the test statistic using the appropriate equation
  - compare calculated value with the critical value
  - if calculated value is greater than the critical value
    - ⇒ result of test is significant
    - ⇒ Data not consistent with initial hypothesis
- Note: A few tests use a 'lower tail' value for which a test statistic below the critical value is significant
  - check the instructions carefully

p-values



p-values

p-value: probability of obtaining a particular value for the test statistic, or a more extreme value, given the null hypothesis



Using p-values

- Some software calculates p-values
  - if p is large (>0.05), the probability of the result occurring by chance is high
    - ⇒ result of test is not significant
  - if p is small (<0.05), the probability of the result occurring by chance is low
    - ⇒ result of test is significant

Essential statistics – Part 2

- Applications of significance tests
  - *t*-tests
  - *F*-test
  - Analysis of variance (ANOVA)

Any questions?

LGC